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THE POSSIBILITY OF A REALIZATION OF FOUR-FOLD SPACE.[1]

ANY magnitude that is a function of a single variable may be represented geometrically by a straight line. Functions of two variables are represented by curved lines or by plane areas; and functions of three variables by either twisted curves, curved surfaces, or volumes. The conceptions of length, area, and volume when used in this way are evidently independent of any of the properties of matter except extension. The question now before us is this, Can we develop and use in a similar way a space-concept which can fully represent a function of four independent variables?

Perhaps most of us can remember times in the course of our education when new conceptions of quantity entered into our conscious life, conceptions which correspond in a general way with those of length, area, and volume, in that they enable us to find at once such relationships as are most frequently required for practical purposes by a general, synthetic, instinctive method. A medical student, instead of memorizing the exact amount of each dose under all possible conditions of the patient, fixes in his mind as in a frame work the medicinal outline of each drug. The student of chemistry does something similar with the elements; the architect has such a concept of structural beauty; the hunter, of the most likely place for game. The sense of propriety, the sense of honor, and numberless other "inbred" or "instinctive" concepts are examples of this mental tendency. There is therefore nothing inherently absurd or improbable in the supposition that any of us may attain to a conception of four-fold space, "as clear as the designer and the draughtsman have of three-fold space."[2] Such a conception would be of great value to all classes of scientists. The biologist could set in this four-fold framework a complete picture of genetic or race relationships; the theologian could use it for the world of spirits; the physicist for forces, etc. By this means ordinary men may become able to see and to develop easily new truths, such as are now revealed only to men of genius and inspiration.

It may be objected that our conception of three-fold space is derived directly from sensations in three-fold space, and that the conception of four-fold space cannot be derived in a similar way, nor yet from sensations in three-fold space. But it is evident that from any sense, from sight, for instance, we get at most a two-dimentional sensation, and it is only by the kind of changes that occur in the sensation that we can infer that a given retinal picture represents extension in two or in three dimensions. In other words, granting, for the sake of the argument, that in sight we perceive directly the existence of two dimensions, it is clear that the existence of a third dimension is solely a matter of inference. It is the simplest hypothesis we can get to explain our sensations. It is conceivable that the hypothesis of a fourth dimension, if it could be made as real to us, might be found of nearly equal value in the simplification of ordinary phenomena. This would be the case if ordinary phenomena involve motion in four independent directions, or if some of the relations of things in the universe, relations not in space, are capable of complete representation in four-fold space. But before we can decide whether or not space and objects of four dimensions exist we must have our ideas of four-fold space developed sufficiently to know what sensations, what visible and tangible phenomena, would be obtained from objects of four dimensions. Up to this time discussions on the reality of four-fold space have been (necessarily) characterized by the absence of evidence for or against.

To develop a clear conception of four-fold space only one course seems to be open, namely, the synthetical study of four-fold geometric figures in the same way that we now study geometric solids. Having given the number and form of the boundaries of a solid we can, by the process of visualization, find more or less easily its appearance (plane projection) in various positions, the possible plane sections, the distance between any two of its points, and so on. In the study of a tessaract (four-fold figure) we should deal similarly with its solid boundaries, finding the possible solid sections, solid projections, and so forth, studying the tessaract by means of conceptions already familiar (length, area, volume), but in new relations. It this way may be developed gradually such a knowledge of the properties of tessaracts as will enable us to "see" them clearly, and to comprehend quickly a new shape. Models of the solid projections and sections are indispensible to rapid progress. Difficulties may, in general, be overcome by considering the analogous difficulties an imaginary plane being, that is to say, a being who has no conception of volume, would have in trying to understand a geometric solid.

The First Lesson.

A point moving in one direction traces a straight line. A line moving perpendicular to itself, in one plane, traces a square; and a square moving similarly traces a cube. How could a plane being learn the number and relations of the faces of a cube? He could readily understand that as the square moves in a direction perpendicular to all of its sides each side traces a new square, and that the moving square in its first and last positions forms the remaining pair of opposite faces. In this way he could count up the six faces, twelve edges, and eight corners of the cube, and might pro-

[1] Digest of a paper read before the Canadian Club of Clark University by T. Proctor Hall, Ph.D

[2] "A New E a of Thought," by C. H. Hinton, M.A.

ceed to make models of the faces as follows (Fig. 1). The side *ab* of the original square *abcd* traces the square *abfe*, which he places, as in the figure, in the only position known to him subject to the condition that *ab* is one of its sides. The three other squares are similarly placed as in the figure, and now five of the six squares are shown in positions which are correct with reference to their generating lines. But the corner *a* is in this figure represented as the generator of two lines *ae*, which is evidently incorrect. The outer squares

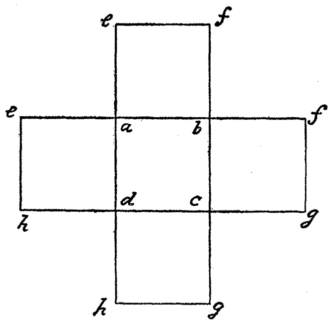

FIG. 1.

are therefore to be turned through 90° about their generating lines until the two lines *ae* become one and the four spaces between *ee*, *ff*, *gg*, *hh*, disappear. He cannot imagine how this is to be done, but he can suppose the central square to move away and disappear in the to him unknown direction, carrying with it the outer squares which would then appear to sink into the centre and disappear as they reached their generating lines until at last the lines *ef*, *fg*, *gh*, *he* reach the position now occupied by the sides of the square *abcd* and become in the picture, what they are really, the sides of the sixth square *efgh*. Supposing, in the next place, that the square *abcd* as it moves away is still visible, but smaller by perspective, the plane being could construct a model which is to us a perspective view of a cube and which would represent to him fairly well the relations of the boundaries of a cube (Fig. 2).

Let us proceed in the same way. A cube moving in a direction perdendicular to all of its faces traces out a rectangular tessaract. Each face traces a new cube, each line a square, each point a line. Counting up we find the tessaract

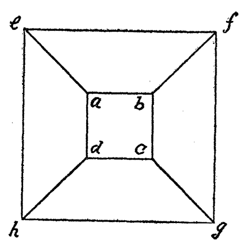

FIG. 2.

is bounded by 8 cubes; has 24 squares, which do not enclose the tessaract, but appear here and there as lines do upon a cube, interfaces, not surfaces; 32 lines or edges, and 16 angular points. A little calculation now shows that each face is common to two cubes, each line to three faces, and each point to four lines. All this seems very abstract, but it becomes real and evident when we make a model. Placing a cube on each face of the original cube, after the analogy of the plane being's squares, we have these six cubes in the only positions known to us which satisfy their genetic conditions (see Fig. 3). The eighth cube is represented by the outer faces of the six cubes, and it is evident that the three lines marked *cC* are really one, the two faces *bC* are one face, and so on. We may now imagine the central cube to move away in the fourth dimension and the others sink inward and disappear as they reach the present boundaries of the central cube, where they turn at a right-angle into the new direction. Finally all the outer faces will meet as the boundaries of the eighth cube *DF*. Supposing the cubes elastic, we may stretch their outer faces and diminish the inner until we obtain the perspective view of a tessaract, as shown in Fig. 4, where the relations of the various boundaries of the tessaract are more easily studied. Incidentally we have learned also that a solid section of the tessaract, when taken parallel to a cube-boundary, is a cube.

The Second Lesson.

Turning again to our imaginary plane being for suggestions, let us see how motion in the third dimension would appear to him. If a cylinder were passing perpendicularly through his plane he would see only a stationary circle, or if it were oblique, a moving ellipse. A cone would appear as a growing or diminishing circle, a beaded rod as an oscillating circle, a corkscrew as an ellipse moving in a circular orbit, and so forth. The stem of a dichotomous tree would be to him a wooden circle which, as the branches approach,

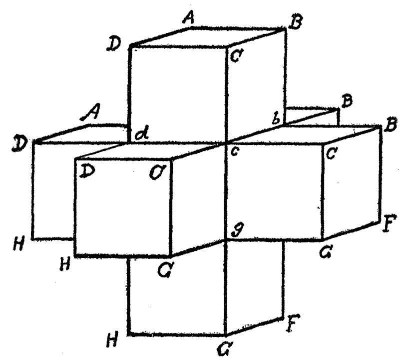

FIG 3.

widens out, becomes constricted, and finally divides into two circles, which repeat the process indefinitely. We may imagine a plane philosopher who, after watching this process for some time, constructs a theory of the evolution of circles. But his idea that all these circles have been developed from one is hardly more than a caricature of the truth.

Every person who has watched the self-division of infusoria under the microscope must be struck with the analogy of these two processes. A little reflection enables us to see that race-unity may be more than a figure of speech or a creation of the fancy; that the organic forms that existed for us yesterday and those that will exist for us to-morrow may be but parts of larger units of which the forms we see to-day are only solid sections. True, this is only a suggestion: but it is a suggestion that carries with it an unavoidable sense of freedom, of fetters loosed, of largeness, and of reality, to anyone who will for a time yield himself to its influence. It is a step toward the poet's view,

> "All are but parts of one stupendous whole,
> Whose body Nature is, and God the soul."

If four-fold space exists, it is evident that it must contain an infinite variety of three-fold spaces, of which we know only one. It must also be everywhere possible for a four-fold being to step out of our space at any point and re-enter

it at any other point; for his relation to our space is nearly the same as our relation to a plane. If ghosts are four-fold beings, the erratic nature of their movements may become more comprehensible in the course of time. An ordinary knot could in four-fold space be readily untied by carrying one loop out of our space and bringing it back in a different place. In fact, a knot in our space would be simply a loop or coil in four-fold space. A flexible closed shell could be turned inside out as easily as a thin hoop can with us: and many other apparent impossibilities become mere child's play. But the realization of four-fold space cannot be learned by giving attention to such little curiosities as these. Only a systematic and continued study of the figures and motions of higher space can be expected to give results of

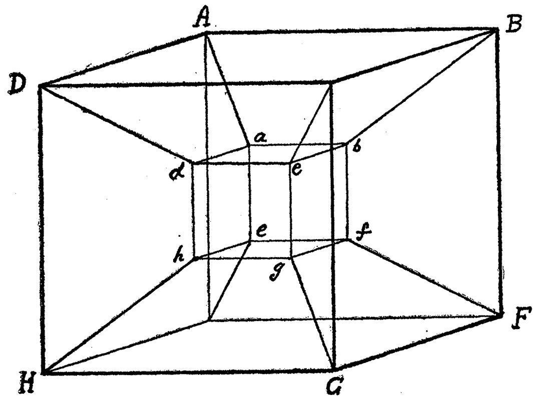

FIG. 4.

educational value. And when (or, if) our conception of four-fold space becomes clear, we shall be ready to recognize the existences and motions of the fourth dimension if there be such.

THE TUSKALOOSA FORMATION.

PROFESSOR LESTER F. WARD has recently spent a couple of weeks in Alabama in making a study of the Tuscaloosa formation, both as to its stratigraphy and its fossil plants. While in Alabama the professor made Tuscaloosa his headquarters, and from there made a number of short excursions in company with Dr. Eugene A. Smith to places of interest. At Cottondale, some eight miles east of Tuscaloosa, there is a fine locality for the collection of fossils, chiefly well-preserved leaf impressions. Professor W. M. Fontaine, a number of years ago, spent some time here and collected a great number of these leaves, which are now in the hands of Professor Ward for study and description.

Between Cottondale and Woodstock there are many occurrences of the Tuscaloosa sands and clays, which are now only outlying remnants, upon the rocks of the Coal Measures, of what was once probably a continuous mantle. Although there are many places where excellent clays for economic purposes are to be seen, none of them thus far examined have been found to contain the leaf impressions. From their position, these beds, occurring between Cottondale and Woodstock, appear to be the oldest of the Tuscaloosa series, and the leaf-bearing beds thus appear to be tolerably well up in the formation, although wherever seen, at Cottondale, Tuscaloosa, Snow's, Shirley's Mill, Glen Allen, etc., the leaf-bearing clays rest directly and unconformably upon the Coal Meas ures, usually within thirty or forty feet of the lin of contact of the two formations.

The other localities mentioned above, except Glen Allen, being away from the railroad lines, had to be reached by private conveyance.

Snow's, about seven or eight miles west of Tuscaloosa, was first examined by Dr. Smith seme years ago, and Professor Fontaine made a large collection here also. In the gullies near Snow's there is fine opportunity for seeing the strata of the Tuscaloosa formation, in vertical section. One of these is more than one hundred feet deep. Shirley's Mill, eleven miles south-east of Fayette Court House, was first made known as a plant-bearing locality by Dr. George Little, who visited it last year while making an examination of the Tuscaloosa clays, for the Geological Survey of the State. Dr. Little bro ght back a few fine leaf impressions from here, but Professor Ward was the first to make a systematic collection of the fossil plants. Glen Allen, on the Kansas City, Memphis, and Birmingham Railroad, was first examined and a small collection made by Dr. Smith several years ago, but here again Professor Ward was the first to collect on a large scale. The leaves are in a dark colored clay that at certain stages of wetness is tough and intractable, but when properly dry yields beautiful specimens at every st. ke of the hammer. The same is true of the clays near Shirley's Mill, and at both these places one can in a few hours load a wagon with fine cabinet specimens.

The Tuscaloosa formation is now generally considered a member of the lower Cretaceous, in part at least equivalent to the Potomac of McGee. While the fossils have not yet been sufficiently studied to decide their exact equivalence, many of the leaves appear to be identical with those occurring in the Amboy clays of New Jersey.

While in Tuscaloosa Professor Ward had an opportunity also of collecting some rare living plants. Upon the banks of the Warrior River, a few miles above the town, under the guidance of Drs. Bondurand and Hall, he was able to obtain *Neviusia Alabamensis*. *SedumNevii*, *Croomia pauciflora*, all comparatively rare, the first named having been found only in this locality. In Dr. Smith's yard is growing the *Croton Alabamensis*, recently discovered on the banks of the Cahaba River, and of interest as being the only shrubby Croton in our North American flora. This one grows to the height of eight or ten feet and makes almost impenetrable thickets. When slightly bruised the leaves and stems give out a fragrance somewhat like that of the flowers of the crab-apple.

An excursion was also made by Dr. Smith and Professor Ward to a little village, Havana, some twenty-five miles south of Tuscaloosa, long known to the former as an interesting locality, where, in a rocky glen under overhanging cliffs, grow two rare ferns, *Asplenium ebenoides* and *Trichomanes radicans*. The former has been noted from only three other localities, all in different States of the Union. Near Havana there are some great gullies, locally known as "The Caves," in which the micaceous sands of the uppermost of the Tuscaloosa formation are laid bare. These sands are remarkable for their brilliant colors, red, pink, purple, and yellow. In this respect they called to mind the similar bright hues of Gay Head in Massachusetts.

E. A. S.

MR. W. J. HUSSEY of the Ann Arbor Observatory has received an appointment as astronomer at the Leland Stanford, Jr., University.